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INVESTIGATION OF FLOW STABILITY IN A CHANNEL WITH A CLOSING SHOCK AT TRANSONIC FLOW VELOCITY

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We study the stability of a flow with a closing shock in a channel, in the case when the flow velocity ahead of the shock is nearly sonic and the "quasi-cylindrical" approximation which was used to analyze the stability in [1, 2] is therefore unsuitable. For this reason we use here the "transonic" approximation which takes into account the variation in the intensity of the acoustic waves as they propagate along the channel. We neglect the variations in the stationary parameters of the flow (which are not equal to the difference $M - 1$, where M is the Mach number) along the channel (from the cross section of the closing shock to the cross section of the channel outlet), and of the derivative of the Mach number with respect to the longitudinal coordinate. The latter situation occurs, in particular, in the vicinity of the minimum section of the Laval nozzle. The remaining formulation of the problem is the same as that given in [2], and includes the condition of reflection at the channel outlet. This condition has the form of a linear relation connecting the nonsteady perturbation of the left Riemann invariant characterizing the reflected acoustic wave, with the perturbations of the right Riemann invariant and entropic function which characterize the waves arriving at the outlet section from the direction of the channel. The " D-subdivision" method [3, 4] is used to construct the region of stability in the plane of the reflection coefficients.

1. Let a steady supersonic flow in a channel, the transverse cross section area of which is $F = F(x)$, where x is the coordinate measured along the channel axis, contain a so-called closing shock situated at $x = 0$. Taking the distance between the section of the shock and the channel outlet as the characteristic length, we find that the outlet section corresponds to $x = 1$. We reduce the equations to their dimensionless form using, as in [2], the critical magnitudes of the steady flow to the left of the shock (the gas moving from the left to right) as its characteristic velocity and density.

Since the formulation of the problem of stability of the stationary flow in question is the same as that in [1, 2], we give, without further ado, the equations and the boundary conditions obtained as the result of linearization. It was shown in [2], that the linearized equations of a one-dimensional, nonstationary flow of a perfect gas, can be written in the following "characteristic" form:

$$\frac{D^+R}{Dt} = a_{11}R + a_{12}L + a_{13}S \tag{1.1}$$

$$\frac{D^-L}{Dt} = a_{21}R + a_{22}L + a_{23}S, \quad \frac{DS}{Dt} = 0$$
$$\frac{D^+}{Dt} = \frac{\partial}{\partial t} + (U + A)\frac{\partial}{\partial x}, \quad \frac{D^-}{Dt} = \frac{\partial}{\partial t} + (U - A)\frac{\partial}{\partial x}$$
$$\frac{D}{Dt} = \frac{\partial}{\partial t} + U\frac{\partial}{\partial x}$$

Here t is time; R, L and S denote the nonstationary perturbations of the left and right Riemann invariants and of the entropy function, respectively; D^+ / Dt, D^- / Dt and D / Dt are the differential operators along the characteristics of the first and second family and of the particle trajectory; U and A are the stationary values of the stream velocity and the speed of sound. The coefficients a_{ij} are known functions of x and are proportional to M', where $M = U / A$ is the Mach number and a prime denotes a derivative with respect to x. For a perfect gas we have

$$M' = M\,[2 + (\varkappa - 1)M^2]\,(\ln F)' \,/\, 2\,(M^2 - 1) \tag{1.2}$$

where $\varkappa$ is the adiabatic ratio.

In accordance with (1.1), the times of propagation of the acoustic and entropic waves (R-, L- and S-waves) between the shock and the channel outlet (or in the opposite direction for the L-waves), are

$$\tau_r = \int_0^1 \frac{dx}{U + A}, \quad \tau_l = \int_0^1 \frac{dx}{A - U}, \quad \tau_s = \int_0^1 \frac{dx}{U} \tag{1.3}$$

In the present case for which $0 < U < A$, i.e. the flow is subsonic when $0 < x < 1$, we have the inequality $\tau_l > \tau_r$.

The system (1.1) must be supplemented by the conditions formulated at the boundaries of the part of the channel in question (at $x = 0$ and $x = 1$). The conditions at $x = 0$ which are obtained by linearizing the relations at the closing compression shock, can be reduced to the following equations [2]:

$$R_+ = \varphi L_+ - \psi Y x_s, \quad S_+ = \varphi' L_+ - \psi' Y x_s, \quad x_s^{\cdot} = \mu L_+ - \beta Y x_s \tag{1.4}$$
$$(Y \equiv (\ln F)'_{x=0} = 2\,(M_+^2 - 1)\,M_+' \,/\, [2 + (\varkappa - 1)\,M_+^2]\,M_+)$$

where $x = x_s(t)$ represents the equation of the trajectory of the closing shock, a dot denotes the differentiation with respect to t, the coefficients φ, ψ, φ', ψ', μ and β are known functions of $\varkappa$ and M_-, and the subscripts minus and plus denote the values of the parameters at $x = 0$ to the left and to the right of the closing shock, while Y is expressed in terms of M_+' in accordance with (1.2).

As in [2], we impose the following condition of reflection at the channel outlet (where $x = 1$):

$$L = \chi R + \chi' S \tag{1.5}$$

Here the reflection coefficients χ and χ' are assumed specified. The condition (1.5) connects the amplitude of the L-wave reflected from the outlet section with the amplitudes of the R- and S-waves arriving at this section.

In [1, 2] the stability was analyzed using the quasi-cylindrical approximation in which the relative changes in the values of the functions R and L during the motion of the corresponding waves from the shock section to the channel outlet section (or in the opposite direction for the L-waves) were neglected. From (1.1) it follows that this occurs in the case of a cylindrical channel when $M' \equiv 0$. In the case of a channel of variable cross section and fixed M_- the error of the approximation in question, decreases as $M' \to 0$. This follows from the fact that, in accordance with what was said before, all coefficients a_{ij} in (1.1) are proportional to M'.

When $M_- \to 1$, the conditions of applicability of the quasi-cylindrical approximation become more and more stringent (in the case of restrictions imposed in the quantity M'), and this is connected with two circumstances. First, the time τ_l of propagation of the L-wave along the channel increases, by virtue of (1.3), without bounds as M_- tends to unity. Consequently, in order that the increase in the value of the left invariant be small compared with $\max(R, L, S)$ when the L-wave moves from the section $x = 1$ to the closing shock, the following condition must hold:

$$\tau_l \max_{0 \leqslant x \leqslant 1,\ j=1,2,3} |a_{2j}| \ll 1 \tag{1.6}$$

In the present case τ_r is of the order of unity, and the coefficients a_{1k} and a_{2j} have the same order of magnitude. Therefore the condition (1.6) or the equivalent inequality $|M'| \ll (M_- - 1)$ ensures that the increments in the values of not only the left, but also of the right invariant, are small compared with $\max(R, L, S)$. This does not however imply the validity of the quasi-cylindrical approximation. In fact, the smallness of the increments in the values of the invariants R and L as compared with $\max(R, L, S)$ implies that they are small compared with their values at $x = 0$, i.e. with R_+ or L_+, only in the case when R_+, L_+ and S_+ are of the same order of magnitude. The possibility of violating this condition represents the second circumstance, and this must be kept in mind when using the quasi-cylindrical approximation. Let us compare the orders of the quantities indicated above in the case when M_- is almost equal to unity. To do this, we rewrite the relations (1.4), taking into account the expressions for the coefficients φ, φ', ... in [2], and retaining in the expansions of these coefficients in terms terms of $\varepsilon \equiv M_- - 1$, only the principal terms. This yields the following expressions:

$$R_+ = -\gamma\varepsilon^2 L_+ + 4(\varkappa + 1)^{-2} M_+' \varepsilon^2 x_s$$
$$S_+ = \varkappa\gamma\varepsilon^2 L_+ + 4\varkappa\gamma(\varkappa + 1)^{-1} M_+' \varepsilon^3 x_s$$
$$x_s^{\cdot} = 0.5(\varkappa + 1) L_+ + 2(\varkappa + 1)^{-1} M_+' \varepsilon x_s \quad (\gamma = 4(\varkappa - 1)/(\varkappa + 1))$$

Integrating the last equation of the above system and restricting ourselves from now on to the case of the negative values of M_+', we find that at sufficiently large t, x_s does not exceed $L_+ / \varepsilon M_+'$ in its order of magnitude. This, together with the first two equations of the system, implies that the maximum possible values of R_+ and S_+ are of the order of εL_+ and $\varepsilon^2 L_+$, respectively. The estimate of the magnitude of S^+ remains valid also when the shock oscillates with a "moderate" or a "high" frequency and x_s has the order of L_+. R_+ however is of the order of $\varepsilon^2 L_+$. From all this and in accordance with (1.1), $\max(R, L, S) = \max L_+$, while R_+ is always smaller than L_+.

Thus the functions R_+, L_+ and S_+ assume different orders of magnitude as $M_- \to 1$. Therefore, to obtain the condition of applicability of the quasi-cylindrical approximation we must inspect the coefficients a_{ij} in more detail. Performing the necessary manipulations we can show that $a_{12} \sim \varepsilon M'$, and the remaining coefficients a_{ij} are of the order of M'. This, and the estimates given above for R_+ and S_+, imply that the variation in the magnitude of the right invariant during the passage of the R-wave from the shock to the channel outlet is of the order of $\varepsilon M' L_+$, and the variation in the magnitude of the left invariant during the passage of the L-wave along the channel in the opposite direction is of the order of $\tau_l M' L_+ \sim M' \varepsilon^{-1} L_+$. Since R_+ can be of the order of $\varepsilon^2 L_+$, the conditions ensuring the smallness of the relative variations in R and L, and consequently the validity of the quasi-cylindrical approximation, can be expressed in the form of the two inequalities: $|\varepsilon M' L_+| \ll |R_+| \sim \varepsilon^2 |L_+|$ and $|M'\varepsilon^{-1} L_+| \ll |L_+|$, which reduce to the following single inequality:

$$|M'| \ll M_- - 1 \qquad (1.7)$$

It was already shown that (1.6) and (1.7) are equivalent by virtue of the fact that the quantities M' and a_{2j} are of the same order.

In accordance with (1.2) $M' \sim F' / (M - 1)$, therefore the quasi-cylindrical approximation can be used only when the derivative F' decreases with $M_- \to 1$ as $(M_- - 1)^2$, and this restricts extraordinarily the feasibility of using this approach. On the other hand, in the case when the velocity ahead of the closing shock is transonic, the author of [5] gives some qualitative considerations on the possible flow instability under the conditions for which the quasi-cylindrical approximation predicts stability. This justifies the development of an approach, the applicability of which to all $M_- > 1$ would be restricted by the inequality $|M'| \ll 1$, the latter being weaker than (1.7). Such an approach which we shall call a "transonic" approximation, is developed below and based on the following concepts.

We limit ourselves to the case in which the gas accelerates in the Laval nozzle ahead of the shock, since for this nozzle M_-, and consequently M_+, can have values as near to unity as required. In the stationary mode a saddle point corresponds to the minimum cross section of the nozzle, and the integral curve corresponding to the accelerating gas represents one of the separatrices of the field of integral curves. Taking this into account, we replace M' for $0 < x < 1$, by the constant M_+', and neglect (by virtue of the assumption that M_+' is small) the variations in the values of all stationary gas parameters except for the differences $A - U$ or $M - 1$ along the channel length. In accordance with the estimates for R_+ and S_+ and the coefficients a_{ij} appearing in the right-hand sides of (1.1) which were given above, only the second terms need be retained when $|M'| \ll 1$ and $(M_- - 1) \ll 1$. Moreover, the corresponding term in the first equation is retained only in the case when $R_+ \sim \varepsilon^2 L_+$, since it is only in such a situation that the increase in the value of the invariant R governed by this term and being of the order of $\varepsilon M' L_+$ is similar to, or even exceeds R_+. If on the other hand $R_+ \sim \varepsilon L_+$, then it is simply immaterial whether this term, and incidentally the neglected term $a_{11} R$ which in this case is similar to $a_{12} L$, are neglected.

In accordance with all that has been said, the flow in question is described, in the transonic approximation, by the following system:

$$\frac{D^- L}{Dt} = aL, \quad \frac{D^+ R}{Dt} = b(1 + \alpha x) L, \quad S(x, t) = S(\zeta) \qquad (1.8)$$

$$\zeta = x - U_+ t, \quad \tau_r = 1 / (U_+ + A_+), \quad \tau_l = \delta^{-1} \ln (1 + \alpha), \quad \tau_s = 1 / U_+$$

$$a = A_+ \left[\frac{1}{2}\left(\frac{1}{M_+} - 1\right) - \frac{(\varkappa - 1) M_+ + 2}{(\varkappa - 1) M_+^2 + 2}\right] M_+'$$

$$b = A_+ \frac{(M_+ - 1)[(\varkappa - 1) M_+^2 - 2]}{2 M_+ [(\varkappa - 1) M_+^2 + 2]} M_+', \quad \alpha = \frac{M_+'}{M_+ - 1} \qquad \delta = -A_+ M_+'$$

The above system was derived using the expressions for a_{12} and a_{22} given in [2].

Here we must stress the fact that the transonic approximation is in fact valid for any M_- when $|M'| \ll 1$. If, however, the value of the Mach number ahead of the shock differs appreciably from unity, then the results of the transonic approximation will be indistinguishable from those of the quasi-cylindrical approximation.

2. It can be shown that, just as in the quasi-cylindrical approximation [1, 2], the analysis of the stability of the flow described by the system (1.8) and the boundary conditions (1.4) and (1.5), can be reduced to the study of the behavior of $L_+(t)$ and $x_s(t)$ as $t \to \infty$. The above functions are defined by a system of two equations. The first of these equations is the third equation of (1.4) rewritten with the arguments included

$$x_s^{\cdot}(t) = \mu L_+(t) - \beta Y x_s(t) \tag{2.1}$$

while the second equation is an integro-difference equation and not a difference equation as in the case studied in [2], and is obtained as follows.

First we find $L = L(x, t)$. Integrating the first equation of (1.8) along the c^--characteristic which passes through the point in question on the xt-plane, we obtain

$$L(x, t) = L_+[t + \tau_l(x)] \exp[-a\tau_l(x)] \tag{2.2}$$

Here $t + \tau_l(x)$ denotes the instant at which the characteristic intersects the closing shock (in the present approximation the characteristic intersects the straight line $x = 0$). Since in the present case $1 - M(x) = (1 - M_+)(1 + \alpha x)$ we have, in accordance with the equation describing the c^--characteristic,

$$\tau_l(x) = \delta^{-1} \ln(1 + \alpha x) \tag{2.3}$$

Now we use (2.2) and (2.3) to integrate the second equation of (1.8) along the c^+-characteristic, from its point of intersection with the straight line $x = 0$ at the time $t - \tau_r(x)$, where $\tau_r(x) = x\tau_r$, to the point with the coordinates x and t. This yields

$$R(x, t) = R_+[t - \tau_r(x)] - b \int_0^{\tau_r(x)} \left[1 + \frac{\alpha\theta}{\tau_r(x)}\right]^{\nu} L_+\left\{t - \tau_r(x) + \theta + \frac{1}{\delta}\ln\left[1 + \frac{\alpha\theta}{\tau_r(x)}\right]\right\} d\theta \quad \left(\nu = 1 - \frac{a}{\delta}\right) \tag{2.4}$$

The formulas (2.2) and (2.4) hold, in particular, in the section of the channel outlet (at $x = 1$). Substituting $R(1, t)$ and $L(1, t)$, and the function $S(1, t)$ which in accordance with (1.8) is equal to $S_+(t - \tau_s')$, into the condition of reflection (1.5) and eliminating R_+ and S_+ from the resulting expression with the help of the first two equations of (1.4), we arrive at the required integro-difference equation

$$L_+(t + \tau_l) e^{-a\tau_l} = \chi\left\{\varphi L_+(t - \tau_r) - \psi Y x_s(t - \tau_r) + \tau_r b \int_0^1 (1 + \alpha x)^{\nu} L_+\left[t - \tau_r + \tau_r x + \frac{1}{\delta}\ln(1 + \alpha x)\right] dx\right\} + \chi'[\varphi' L_+(t - \tau_s) - \psi' Y x_s(t - \tau_s)] \tag{2.5}$$

When $a = b = 0$, Eq. (2.5) becomes a corresponding difference equation of the quasi-cylindrical approximation [2].

The characteristic equation of the system (2.1) and (2.5) the roots λ of which determine the behavior of the eigenfunctions $L_+(t) = L^\circ \exp \lambda t$ and $x_s(t) = X^\circ \exp \lambda t$ where L° and X° are constants, and therefore also the evolution of the solution at large t, has the form

$$H(\lambda) \equiv (\lambda + \beta Y)e^{-a\tau_l} - \chi[\varphi(\lambda + \beta Y) - \mu\psi Y + \tau_r b(\lambda + \beta Y) I(\lambda)]e^{-\lambda\tau} - \chi'[\varphi'(\lambda + \beta Y) - \mu\psi' Y]e^{-\lambda\tau'} = 0 \tag{2.6}$$

$$I(\lambda) = \int_0^1 (1 + \alpha x)\exp[\lambda(x-1)\tau_r + (\lambda - a)\tau_l(x)]\,dx$$

$$\tau = \tau_l + \tau_r, \quad \tau' = \tau_l + \tau_s$$

The integral $I(\lambda)$ can be expressed in terms of the degenerate hypergeometric functions. Performing the necessary manipulations which include the use of (2.3) and of the Kummer's formula [6]: $\Phi(\alpha, \gamma; r) = \Phi(\gamma - \alpha, \gamma; -r)\exp r$, where $\Phi(\alpha, \gamma; r)$ is a degenerate hypergeometric function, we can show that

$$I(\lambda) = \frac{\delta}{\alpha(2\delta - a + \lambda)}\left[(1+\alpha)^2 e^{\lambda\tau - a\tau_l}\,\Phi\left(1, 3 + \frac{\lambda - a}{\delta}; -\lambda\tau_r\frac{1+\alpha}{\alpha}\right) - \Phi\left(1, 3 + \frac{\lambda - a}{\delta}; -\frac{\lambda\tau_r}{\alpha}\right)\right] \tag{2.7}$$

In addition to the results obtained by investigating the characteristic equation (2.6) with the function $I(\lambda)$, given by (2.7), we have also obtained the proof, as we shall see below, for a somewhat simpler method of approximate computation of the integral $I(\lambda)$ and this now follows. The difficulty encountered in computing $I(\lambda)$ is caused by the complexity of the function $\tau_l(x)$ given by (2.3) and such that $\tau_l(0) = 0$ and $\tau_l(1) = \tau_l$. If we replace the curve (2.3) by the straight line $\tau_l(x) = \tau_l x$, then simple transformations yield

$$I(\lambda) = \gamma^{-2}\{\alpha - \gamma + [(1+\alpha)\gamma - \alpha]\,e^{\gamma}\}\,e^{-\lambda\tau_r} \tag{2.8}$$
$$\gamma = \gamma(\lambda) = \lambda\tau_r + (\lambda - a)\tau_l$$

As we have already said, the study of stability is based on constructing the Nyquist curves (the " D -subdivision" method) and on asymptotic analysis valid for $|\lambda| \gg 1$. Without going into details which are, on the whole, the same as those in [2], we shall indidicate just three aspects.

First we shall deal with the position of the region of stability which is represented by the $D(0)$ region in the D-subdivision method. Setting in (2.6) $\chi = \chi' = 0$, we arrive at a characteristic equation with a single root $\lambda = -\beta Y$. Computations in [1] show that β is always positive. From this it follows that when the channel widens $(Y > 0)$, the region of stability in the $\chi\chi'$-plane occupies a neighborhood of the coordinate origin.

The second aspect is connected with the asymptotic analysis of the roots of the characteristic equation when $|\lambda| \gg 1$. This analysis is carried out in the same manner as in the quasi-cylindrical approximation [2] and shows, that when $|\lambda| \gg 1$, the region of stability is a rhombus

$$|\chi\varphi \pm \chi'\varphi'| < e^{-a\tau_l}$$

The last aspect concerns the separation of the real and imaginary parts of $H(\lambda)$, which is necessary for the construction of the Nyquist curves. Such separation was performed

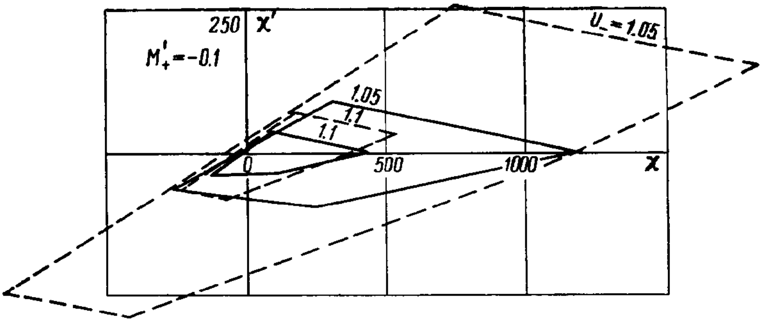

Fig. 1

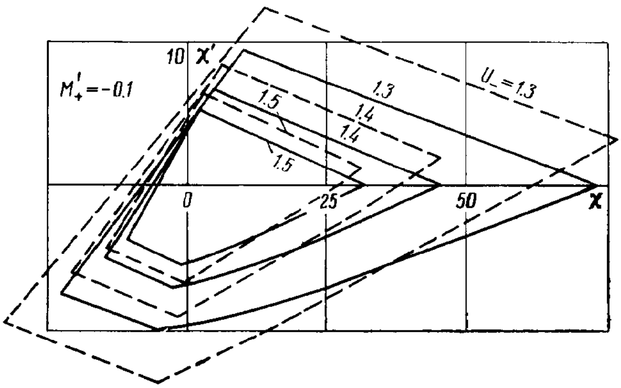

Fig. 2

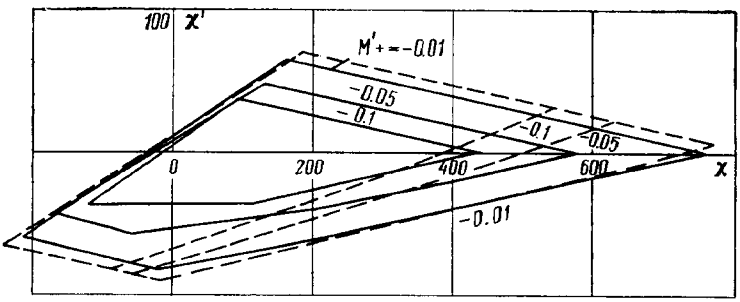

Fig. 3

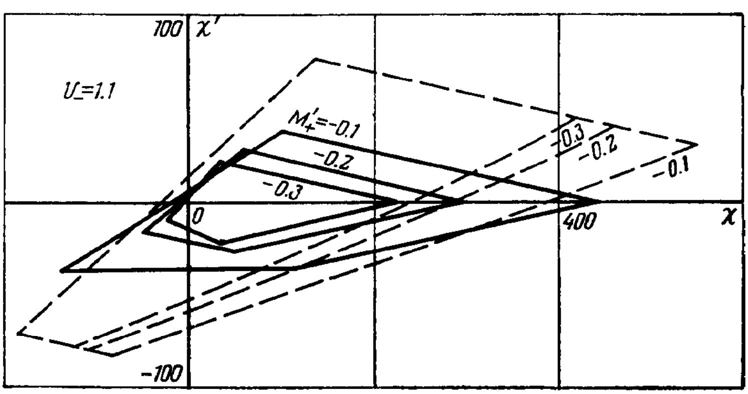

Fig. 4

on a computer, and FORTRAN was used for programing, which was essential in view of the complexity of the functions appearing in $H(\lambda)$.

3. Let us pass to the examples of constructing the regions of stability in the $\chi\chi'$-plane of the reflection coefficients. Some of the results are shown on Figs. 1 — 4 where the solid lines denote the boundaries of the regions of stability obtained in the transonic approximation, and the dashed lines refer to the quasi-cylindrical approximation. All computations were carried out for a flow of a perfect gas with $\varkappa = 1.4$. Figures 1 and 2 show the boundaries of the region of stability in the $\chi\chi'$-plane for $M_+' = -0.1$ and for various values of U_-, i.e. the planes $U_- = \text{const}$ intersect the surface defining the region of stability in the $U_-\chi\chi'$-space, each section denoted by the corresponding value of U_-. The flow is stable for χ and χ' corresponding to the points inside the relevant polygons. Similarly, Figs. 3 and 4 show the boundaries of the regions of stability for $U_- = 1.1$, plotted for various values of M_+' which again accompany the corresponding curves. On inspecting Figs. 1 — 4 we see, that in both approximations, transonic and quasi-cylindrical, the size of the region of stability increases, for a fixed M_+', as U_- approaches unity. As expected, the difference between the solid and the dashed lines increases with decreasing U_- and increasing M_+'. For a fixed M_+' we see, that the increase in the value of U_- is accompanied by a spontaneous transition from the results of the transonic approximation, to the results of the quasi-cylindrical approximation. The same happens if we fix U_- and decrease the value of M_+'. We note that the results depicted on Figs. 1 — 4 by the solid lines, are obtained with the use of the formula (2.7). The results obtained for the approximate formula (2.8) coincide with the previous ones within the range of the graphical representation, for the values of $\alpha < 4$. The divergence however increases with increasing α and becomes very noticeable when $\alpha \geqslant 6$.

From the data shown above it follows that athough the numerical results of the quasi-cylindrical and the transonic approximation diverge from each other when the value of the Mach number ahead of the closing shock tends to unity, the fundamental arguments made in [1, 2] within the framework of the quasi-cylindrical approximation, remain unchanged. In particular, we retain the arguments concerning the stability of a flow with a closing shock in a diverging section of the channel $(Y < 0,\ M_+' > 0)$ provided that the conditions of the absence of reflection, constancy of pressure or constancy of the Mach number are prescribed at the channel outlet. Since the transonic approximation is based on the smallness of the derivative $M_+' = (dM / dx)_{x=0}$, where the distance between the shock and the channel outlet is taken as the unit length, it follows that it can be used e.g. in the case when the shock lies near the outlet section. In this connection we note, that the tendency to instability which was discovered for the present case in [5], was caused by the improper manner of neglecting the nonlinear terms in the Taylor expansion in x of the solution.

We conclude with an important remark. When the equations are linearized, the coefficient $(u - a)$ accompanying $\partial L / \partial x$ is replaced by $U - A$. When the flow is transonic, such a substitution is valid only in the case when the perturbations in the values of the corresponding parameters are small compared with the difference $U - A$. This imposes a very rigid limitation on the application of the transonic approximation in the cases when the perturbation amplitude is determined by the conditions of the problem. Investigation of the stability however is a different matter. Here we have to analyze the behavior of arbitrarily small perturbations the amplitude of which, at $t = 0$, can be chosen to correspond, in particular, to the conditions of applicability of the equations used. If the flow is stable, then the perturbations will not increase and the conditions indicated will also be observed at $t > 0$. Thus the arguments concerning the stability in its classical sense made on the basis of utilizing Eqs.(1.1) or (1.8) are valid outside the range of their dependence on the magnitude of the difference $U - A$, provided that the latter does not vanish at $0 < x < 1$.

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NONLINEAR PROPAGATION OF WAVES IN MEDIA WITH AN ARBITRARY NUMBER OF CHEMICAL REACTIONS

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We consider nonlinear wave motions in chemically active, gaseous mixtures the change in the composition of which is governed by an arbitrary number of reactions taking place. We impose on the equations of state the conditions ensuring that the frozen and the equilibrium speed of sound have similar values. We carry out an asymptotic analysis of the initial system of Euler equations together with the chemical reaction equations. As the result, we obtain an approximate system of equations for the velocity of the medium particles, and for the reaction completeness vector the order of which is equal to the number of the relaxation processes plus one.

1. Thermodynamics of the system. We assume that N reactions take place in the flow of a chemically active gaseous mixture. The change in the composi-